Butterworth Filter:

$f_p = 20kHz,\ f_s = 60kHz, R_p = 0.1dB, R_s = 50dB$

Selective parameter: $k = \frac{f_p}{f_s} = \frac{1}{3}$

Discrimination parameter: $k_1 = \sqrt{\frac{10^{R_p/10}-1}{10^{R_s/10}-1}} = \sqrt{\frac{10^{0.1/10}-1}{10^{50/10}-1}} \approx 0.000483$

$$n \geq \frac{|\log k_1|}{|\log k|} = \frac{\log 1/k_1}{\log 1/k} = \frac{\log 1/0.000483}{\log 3} \approx 6.95$$

So, the minimum order of Butterworth filter that can meet the requirements is 7.

$$\omega_0 = \omega_s \left(10^{\frac{R_s}{10}} - 1\right)^{-\frac{1}{2n}} = 2\pi \cdot 60 \cdot 10^3 \cdot \left(10^{\frac{50}{10}} - 1\right)^{-\frac{1}{2*7}} = 1.6565 * 10^5 rad/s$$

$$C_n = \frac{10^{\frac{R_s}{10}} - 1}{(2\pi \cdot 60 \cdot 10^3)^{2*7}}$$

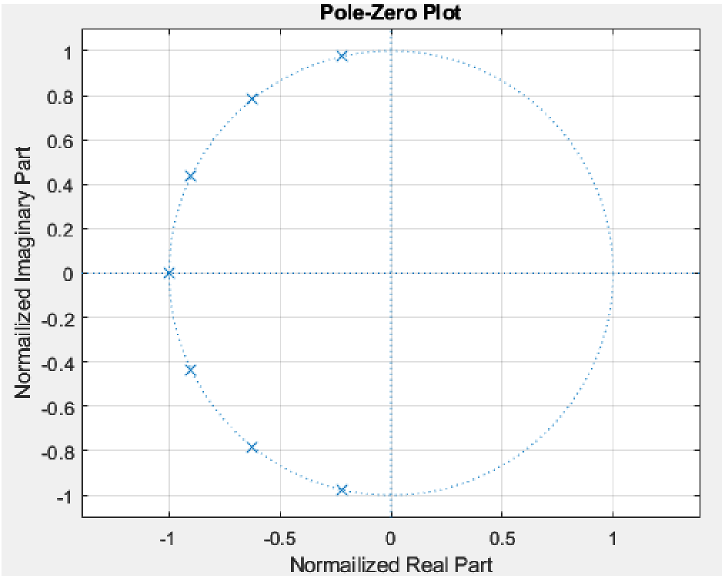
Pole locations (natural modes):

$$s_k = C_n^{-1/2n} e^{j\pi(n-1+2k)/2n}, n = 7, k = 1,2, \dots ,7$$

Poles are: $(-0.3686 \pm 1.615i) * 10^5 rad/s$, $(-1.0328 \pm 1.2915i) * 10^5 rad/s$

and $-1.6565 * 10^5 rad/s$; all zeros are at infinity.

The following plot shows normalized pole location (s_k/C_n).



Chebyshev Filter:

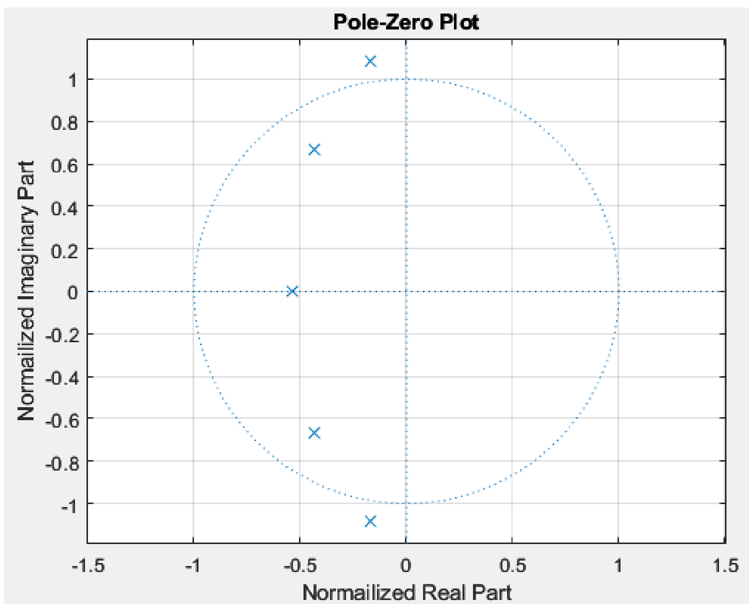
Selective parameter: $k = \frac{1}{3}$, Discrimination parameter: $k_1 \approx 0.000483$

$$n \geq \frac{\cosh^{-1} \frac{1}{k_1}}{\cosh^{-1} \frac{1}{k}} \approx 4.72$$

So, the minimum order of Chebyshev filter that can meet the requirements is 5.

Chebyshev **Type I**: (Refer to P155-P160 for detailed calculation)

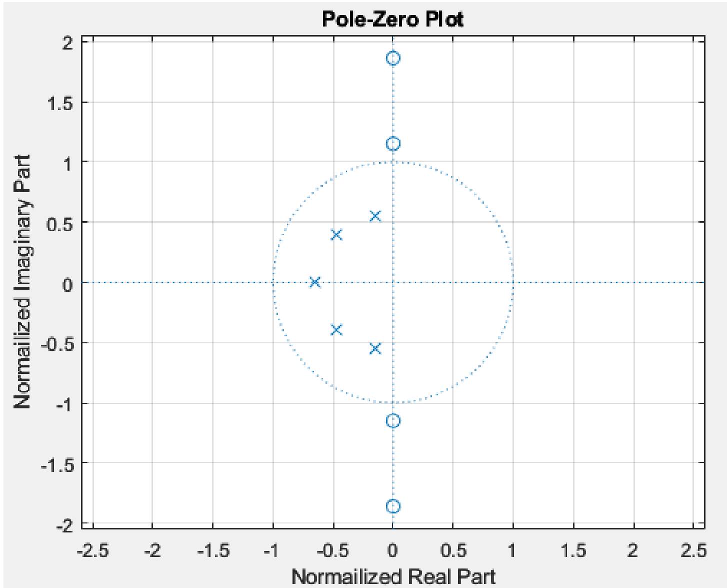
Poles are: $(-0.2093 \pm 1.3576i) * 10^5 rad/s$, $(-0.5479 \pm 0.8391i) * 10^5 rad/s$ and $-0.6772 * 10^5 rad/s$; all zeros are at infinity.



Chebyshev **Type II**:

Poles are: $(-0.5254 \pm 1.8821i) * 10^5 rad/s$, $(-1.6185 \pm 1.3688i) * 10^5 rad/s$ and $-2.2459 * 10^5 rad/s$

Zeros are: $\pm 6.4138i * 10^5 rad/s$, $\pm 3.9639i * 10^5 rad/s$ and one zero is at infinity.

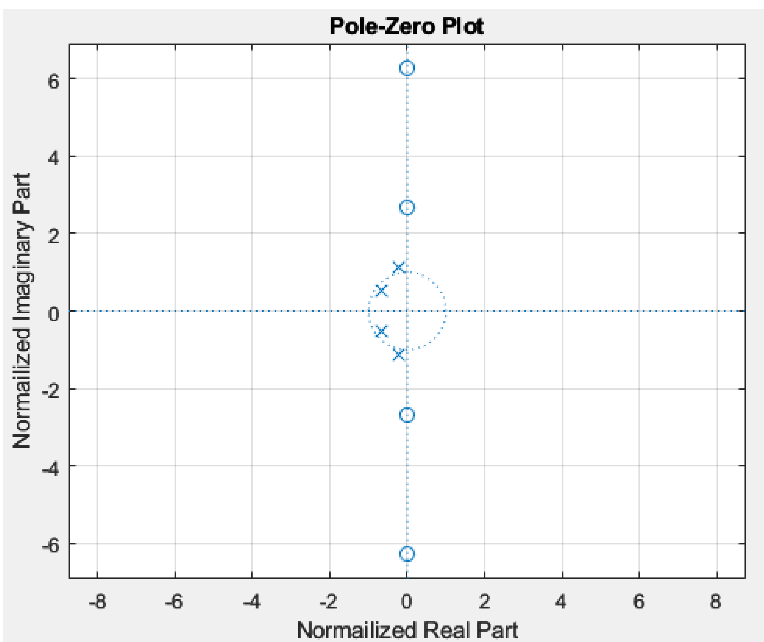


Elliptic Filter:

Minimum filter order is found to be 4 based on matlab function ellipord().

Poles are: $(-0.8272 \pm 0.6375i) * 10^5 rad/s$ and $(-0.2941 \pm 1.4067i) * 10^5 rad/s$

Zeros are: $\pm 7.882i * 10^5 rad/s$, and $\pm 3.3667i * 10^5 rad/s$



Appendix:

```
%% Butterworth Filter
clear
close all
clc
Wp = 20e3*2*pi;
Ws = 60e3*2*pi;
Rp = 0.1;
Rs = 50;

[N,Wn] = buttord(Wp,Ws,Rp,Rs,'s');
[num, den] = butter(N, Wn, 'low', 's');
[z,p,k] = butter(N, Wn, 'low', 's');
zplane(z/Wn,p/Wn);
grid on;
xlabel("Normailized Real Part");
ylabel("Normailized Imaginary Part");


w = logspace(log10(Wp/10),log10(Wn*10),1000);
[H, wout] = freqs(num, den, w);
% Magnitude response
figure(2);
semilogx(w/(2*pi), 20*log10(abs(H)));
grid on
xline(20e3,'-.');
xline(60e3,'-.');
title('Gain Response');
xlabel('Frequency(Hz)');
ylabel('Gain(dB)');
xlim([2e3 6e5]);
ylim([-100,10]);

% Phase response
figure(3);
% semilogx(w, 180/pi*phase(H));
semilogx(w/(2*pi), 180/pi*unwrap(angle(H)));
grid on
title('Phase Response');
xlabel('Frequency(Hz)');
ylabel('Phase(^{o})');
xlim([2e3 6e5]);
```

```matlab
%% Chebyshev Type I
clear
close all
clc
Wp = 20e3*2*pi;
Ws = 60e3*2*pi;
Rp = 0.1;
Rs = 50;

% N = ceil(acosh(sqrt(((10^(50/20))^2-1)/(10^(0.1/20)^2-1)))/acosh(3));
[N,Wn] = cheb1ord(Wp,Ws,Rp,Rs,'s');
[num, den] = cheby1(N, Rp, Wp, 'low', 's');
[z, p, k] = cheby1(N, Rp, Wp, 'low', 's');
zplane(z/Wn,p/Wn);
grid on;
xlabel("Normailized Real Part");
ylabel("Normailized Imaginary Part");


w = logspace(log10(Wp/10),log10(Wn*10),1000);
[H, wout] = freqs(num, den, w);
% Magnitude response
figure(2);
semilogx(w/(2*pi), 20*log10(abs(H)));
grid on
xline(20e3,'-.');
xline(60e3,'-.');
title('Gain Response');
xlabel('Frequency(Hz)');
ylabel('Gain(dB)');
xlim([2e3 6e5]);
ylim([-100,10]);

% Phase response
figure(3);
% semilogx(w, 180/pi*phase(H));
semilogx(w/(2*pi), 180/pi*unwrap(angle(H)));
grid on
title('Phase Response');
xlabel('Frequency(Hz)');
ylabel('Phase(^{o})');
xlim([2e3 6e5]);
```

```
%% Chebyshev Type II
clear
close all
clc
Wp = 20e3*2*pi;
Ws = 60e3*2*pi;
Rp = 0.1;
Rs = 50;

[N,Wn] = cheb2ord(Wp,Ws,Rp,Rs,'s');
[num, den] = cheby2(N, Rs, Ws, 'low', 's');
[z, p, k] = cheby2(N, Rs, Ws, 'low', 's');
zplane(z/Wn,p/Wn);
grid on;
xlabel("Normailized Real Part");
ylabel("Normailized Imaginary Part");


w = logspace(log10(Wp/100),log10(Wn*100),1000);
[H, wout] = freqs(num, den, w);
% Magnitude response
figure(2);
semilogx(w/(2*pi), 20*log10(abs(H)));
grid on
xline(20e3,'-.');
xline(60e3,'-.');
title('Gain Response');
xlabel('Frequency(Hz)');
ylabel('Gain(dB)');
xlim([2e3 6e5]);
ylim([-100,10]);

% Phase response
figure(3);
% semilogx(w, 180/pi*phase(H));
semilogx(w/(2*pi), 180/pi*unwrap(angle(H)));
grid on
title('Phase Response');
xlabel('Frequency(Hz)');
ylabel('Phase(^{o})');
xlim([2e3 6e5]);
```

```
%% Elliptic
clear
close all
clc
Wp = 20e3*2*pi;
Ws = 60e3*2*pi;
Rp = 0.1;
Rs = 50;

[N,Wn] = ellipord(Wp,Ws,Rp,Rs,'s');
[num, den] = ellip(N, Rp, Rs, Wp, 'low', 's');
[z, p, k] = ellip(N, Rp, Rs, Wp, 'low', 's');
zplane(z/Wn,p/Wn);
grid on;
xlabel("Normailized Real Part");
ylabel("Normailized Imaginary Part");


w = logspace(log10(Wp/100),log10(Wn*100),1000);
[H, wout] = freqs(num, den, w);
% Magnitude response
figure(2);
semilogx(w/(2*pi), 20*log10(abs(H)));
grid on
xline(20e3,'-');
xline(60e3,'-');
title('Gain Response');
xlabel('Frequency(Hz)');
ylabel('Gain(dB)');
xlim([2e2 6e6]);
ylim([-100,10]);

% Phase response
figure(3);
% semilogx(w, 180/pi*phase(H));
semilogx(w/(2*pi), 180/pi*unwrap(angle(H)));
grid on
title('Phase Response');
xlabel('Frequency(Hz)');
ylabel('Phase(^{o})');
xlim([2e2 6e6]);
```